NORMAL DISTRIBUTION

p.d.f of a normal distribution is given as

(1) If z is a standard normal variate then find
   a. P(z<1.5)
   b. P(z>1.5)
   c. P(z<-1.5)
   d. P(z>-1.5)

(2) If z is a standard normal variate then find P(-1.3< z< 0.5)

(3) In a sample of 1000 cases, the mean of a certain test is 14 & standard deviation is 2.5. assuming the distribution to be normal find,
   a. How many students score between 12 and 15
   b. How many score above 18
   c. How many score below 8
   d. How many score 16

(4) In the normal distribution 30% items are under 45 & 8% are above 64. Find the mean & standard deviation of the normal distribution.

(5) In a certain examination the percentage of passes & distinction are 45 & 9 respectively. Find the average marks obtained by the candidate & the standard deviation of the distribution, the minimum marks for pass & distinction being 40 & 75 respectively.

(6) Find the mean & standard deviation of the following

$$Ce^{\frac{-1(x^2-6x+4)}{24}}$$

(7) A normal distribution has a mean m & standard deviation σ. What proportion of the population is contained in the range
   a. m±σ (b)m±2σ (c)m±3σ (d)m – σ to m+3σ (e)-∞ to m+σ

(8) Find the equation of best fit normal curve to the following:

x	1	3	5	7	9
f	1	2	3	2	1

(9) In a certain examination, the marks are found to be approximately normally distributed with a mean of 65 & a standard deviation of 10. Find the probability that an individual picked at random, has a mark from 60 to 69 inclusive. If the people with the highest 5% marks are awarded a distinction, find the lowest mark required to obtain a distinction.

(10) A coin is tossed 500 times. Find the probability of getting more than 280 heads.

(11) In a large population, the probability than an individual has characteristic A is 0.8. Find the smallest number of people that must be chosen from this population so that the probability that more than 70% of them have characteristic A is greater than 0.9.

(12) A corner shop stocks fresh milk. The number of bottles of milk sold each day is a Poisson variate with mean of 25 bottles a day. How many bottles of milk should the shopkeeper stock each day so that the probability that we can meet the demand is greater than 0.95?

(13) X is normally distributed & the mean of X is 12 & S.D. is 4.

a. Find out the probability of the following
   i. $X \geq 20$ [Ans: 0.0228]
   ii. $X \leq 20$ [Ans: 0.9722]
   iii. $0 \leq X \leq 12$ [Ans: 0.49865]
b. Find x', where $P(X > x')=0.24$ [Ans: 14.84]
c. Find x_0' and x_1', where $P(x_0' < X < x_1')=0.50$ & $P(X > x_1')=0.25$ [Ans: 9.32; 14.68]

(14) X is a normal variate with mean 30 & S.D. 5. Find the probabilities that

a. $26 \leq X \leq 40$ [Ans: 0.7653]
b. $X \geq 45$ [Ans: 0.00135]
c. $|X - 30| > 5$ [Ans: 0.3174]

(15) The mean yield for one – acre plot is 662kilos with an s.d. 32kilos. Assuming normal distribution, how many one-acre plots in a batch of 1,000 plots would you expect to have yield

a. Over 700kilos, [Ans: 117]
b. Below 650kilos [Ans: 352]
c. What is the lowest yield of the best 100 plots? [Ans: 702.96kilos]

(16) There are six hundred Economics students in the post – graduate classes of a university, and the probability for any student to need a copy of a particular book from the university library on any day is 0.05. How many copies of the book should be kept in the university library so that the probability may be greater than 0.90 that none of the students needing a copy from the library has to come back disappointed? (Use normal approximation to the binomial distribution.) [Ans: 37 copies]

(17) The local authorities in a certain city install 10,000 electric lamps in the streets of the city. If these lamps have an average life of 1,000 burning hours with a standard deviation of 200 hours, assuming normality, what number of lamps might be expected to fail
a. In the first 800 burning hours [Ans: 1,587]
b. Between 800 and 1,200 burning hours [Ans: 6,826]
After what period of burning hours would you expect that
(a) 10% of the lamps would fail [Ans: 744 hours]
(b) 10% of the lamps would be still burning [Ans: 1,256 hours]

(18) The marks obtained by a number of students for a certain subject are assumed to be approximately normally distributed with mean value 65 & with a standard deviation of 5. If 3 students are taken at random from this set, what is the probability that exactly 2 of them will have marks over 70? [Ans: 0.06357]

(19) If $\log_{10}X$ is normally distributed with mean 4 & variance 4, find the probability of $1.202<X<83180000$. [Ans:0.9500]

(20) In a distribution exactly normal, 10.03% of the items are under 25 kilogram weight & 89.97% of the items are under 70 kilogram weight. What are the mean & standard deviation of the distribution? [Ans: 47.5; 17.578]

(21) If the skulls are classified as A, B and C according as the length-breadth index is under 75, between 75 and 80, or over 80, find approximately (assuming that the distribution is normal) the mean & standard deviation of a series in which A are 58%, B are 38% and C are 4%, being given that if

$$f(x)=\frac{1}{\sqrt{2\pi}}\int e^{-x^2/2}\,dx$$

, then f(0.20)=0.08 and f(1.75)=0.46

[Ans: 74.35; 3.23]

(22) In an experiment it is laid down that a student passes if he secures 30 per cent or more marks. He is placed in the first, second or third division according as he secures 60% or more marks, between 45% and 60% marks & marks between 30% and 45% respectively. He gets distinction in case he secures 80% or more marks. It is noticed from the result that 10% of the students failed in the examination, whereas 5% of them obtained distinction. Calculate the percentage of students placed in the second division (Assume normal distribution of marks). [Ans: 0.34]

(23) A sample of 100 items is taken at random from a batch known to contain 40% defectives. What is the probability that the sample contains:
a. At least 44 defectives [Ans: 0.2376]
b. Exactly 44 defectives [Ans: 0.0576]

TABLE 9·2. AREAS UNDER NORMAL CURVE

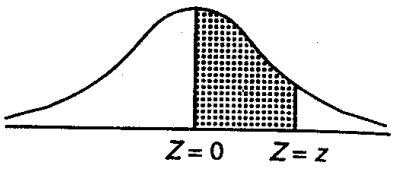
Normal probability curve is given by :

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\} \quad -\infty < x < \infty$$

and standard normal probability curve is given by :

$$\phi(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}z^2\right), \quad -\infty < z < \infty$$

where $Z = \frac{X - E(X)}{\sigma_X} \sim N(0, 1)$



The following table gives the shaded area in the diagram, *viz.*, $P(0 < Z < z)$ for differen values of z.

TABLE OF AREAS

↓Z→	0	1	2	3	4	5	6	7	8	9
·0	·0000	·0040	·0080	·0120	·0160	·0199	·0239	·0279	·0319	·0359
·1	·0398	·0438	·0478	·0517	·0557	·0596	·0636	·0675	·0714	·0759
·2	·0793	·0832	·0871	·0910	·0948	·0987	·1026	·1064	·1103	·1141
·3	·1179	·1217	·1255	·1293	·1331	·1368	·1406	·1443	·1480	·1517
·4	·1554	·1591	·1628	·1664	·1700	·1736	·1772	·1808	·1844	.1879
·5	·1915	·1950	·1985	·2019	·2054	·2088	·2123	·2157	·2190	·2224
·6	·2257	·2291	·2324	·2357	·2389	·2422	·2454	·2486	·2517	·2549
·7	·2580	·2611	·2642	·2673	·2703	·2734	·2764	·2794	·2823	·2852
·8	·2881	·2910	·2939	·2967	·2995	·3023	·3051	·3078	·3106	·3133
·9	·3159	·3186	·3212	·3238	·3264	·3289	·3315	·3340	·3365	·3389
1·0	·3413	·3438	·3461	·3485	·3508	·3531	·3554	·3577	·3599	·3621
1·1	·3643	·3655	·3686	·3708	·3729	·3749	·3770	·3790	·3810	·3830
1·2	·3849	·3869	·3888	·3907	·3925	·3944	·3962	·3980	·3997	·4015
1·3	·4032	·4049	·4066	·4082	·4099	·4115	·4131	·4147	·4162	·4177
1·4	·4192	·4207	·4222	·4236	·4251	·4265	·4279	·4292	·4306	·4319
1·5	·4332	·4345	·4357	·4370	·4382	·4394	·4406	·4418	·4429	·4441
1·6	·4452	·4463	·4474	·4484	·4495	·4505	·4515	·4525	·4535	·4545
1·7	·4554	·4564	·4573	·4582	·4591	·4599	·4608	·4616	·4625	·4633
1·8	·4641	·4649	·4656	·4664	·4671	·4678	·4686	·4693	·4699	·4706
1·9	·4713	·4719	·4726	·4732	·4738	·4744	·4750	·4756	·4761	·4767
2·0	·4772	·4778	·4783	·4788	·4793	·4798	·4803	·4808	·4812	·4817
2·1	·4821	·4826	·4830	·4834	·4838	·4842	·4846	·4850	·4854	·4857
2·2	·4861	·4864	·4868	·4871	·4875	·4678	·4881	·4884	·4887	·4890
2·3	·4893	·4896	·4898	·4901	·4904	·4906	·4909	·4911	·4913	·4916
2·4	·4918	·4920	·4922	·4925	·4927	·4929	·4931	·4932	·4934	·4936
2·5	·4938	·4940	·4941	·4943	·4945	·4946	·4948	·4959	·4951	·4952
2·6	·4953	·4955	·4956	·4957	·4959	·4960	·4961	·4962	·4963	·4964
2·7	·4965	·4966	·4967	·4968	·4969	·4970	·4971	·4972	·4973	·4974
2·8	·4974	·4975	·4976	·4977	·4977	·4978	·4979	·4979	·4980	·4981
2·9	·4981	·4982	·4982	·4983	·4984	·4984	·4985	·4985	·4986	·4986
3·0	·4987	·4987	·4987	·4988	·4988	·4989	·4989	·4989	·4990	·4990
3·1	·4990	·4991	·4991	·4991	·4992	·4992	·4992	·4992	·4993	·4993
3·2	·4993	·4993	·4994	·4994	·4994	·4994	·4994	·4995	·4995	·4995
3·3	·4995	·4995	·4995	·4996	·4996	·4996	·4996	·4996	·4996	·4997
3·4	·4997	·4997	·4997	·4997	·4997	·4997	·4997	·4997	·4997	·4998
3·5	·4998	·4998	·4998	·4998	·4998	·4998	·4998	·4998	·4998	·4998
3·6	·4998	·4998	·4999	.4999	·4999	·4999	·4999	·4999	·4999	·4999
3·7	·4999	·4999	·4999	·4999	·4999	·4999	·4999	·4999	·4999	·4999
3·9	·5000	·5000	·5000	·5000	·5000	·5000	·5000	·5000	·5000	·5000